AIAA 81-0055R

Development of Confined Laminar Wakes at Large Reynolds Numbers

A. Plotkin*
University of Maryland, College Park, Md.

A spectral method (with the streamwise coordinate as the time-like variable) is investigated for calculating the development of the wake flowfield behind a symmetric cascade-like arrangement of bodies for steady two-dimensional laminar incompressible flow at large Reynolds number. The flow is modeled by the slender channel approximation to the Navier-Stokes equations which is boundary-layer-like in character. The stream function is expanded in a Fourier series in the transverse coordinate and the problem is reduced to that of solving a coupled set of quasilinear first-order ordinary differential equations. The method is applied to the separated flow past a cascade of finite thickness flat plates.

Introduction

ONE of the most active areas of current aerodynamic research is the modeling, both analytical and numerical, of high Reynolds number separated flow. The difficulty in obtaining solutions to the complete Navier Stokes equations for such flows has led to the development of a number of techniques based on the solution of a modified set of equations which, for many applications, is boundary-layer-like in character.

Davis and co-workers have carefully studied and compared a variety of these techniques for modeling flows with small separation bubbles. The numerical solution of the Navier-Stokes equations, "parabolized" versions of these equations, interacting boundary-layer theory and asymptotic triple deck theory are discussed for steady two-dimensional laminar separated flow at high Reynolds number in Davis[1] and Davis and Werle.[2] The importance of a combined numerical and analytical approach is emphasized.

The asymptotic high Reynolds structure of a wide variety of flows in the neighborhood of separation has been revealed through the triple deck theory described in Stewartson.[3] The theory provides the appropriate scaling in the separation region and shows that the basic governing equations are those of boundary-layer theory with displacement thickness interaction. The appropriate inviscid outer flow for bluff body separation is postulated to be the Kirchoff free streamline model proposed by Sychev[4] and the combination of the triple deck with this model is used to describe the flow past a circular cylinder by Smith.[5]

The analyses that use the boundary-layer equations to describe the essential features in the neighborhood of separation require a numerical technique to integrate these parabolic equations through a region of reversed flow. The streamwise pressure gradient cannot be specified in this region and must be determined as part of an interaction with the outer inviscid flow. Finite-difference techniques applicable to regions of reversed flow are discussed in Cebeci et al.[6]

In a wide variety of two-dimensional, laminar, incompressible channel flows, the slender channel approximation[7] is valid at large Reynolds numbers. The resulting slender channel equations are identical in form to the boundary-layer equations (with different scaling) but apply to a fully viscous flow. Also, the pressure is an unknown. Some recent solutions for flows without separation are given in Blottner.[8] Smith and Eagles[9] calculated a finite-difference solution to the flow in a slowly diverging channel with separation bubbles of limited extent on the walls.

Kumar and Yajnik[10] solve the slender channel equations in stream function form for a number of separated channel flows including a sudden expansion. They depart from most previous investigators in both their choice of these parabolic equations for modeling flows with extensive recirculation zones and in the choice of a semianalytic spectral or integral relations method for the solution. The stream function is expanded in a series of the far wake eigenfunctions. The problem is reduced to the integration of a coupled set of quasilinear first-order ordinary differential equations. With the use of at most five terms in the series, results are obtained that agree well with comparable numerical solutions of the Navier-Stokes equations.

In this paper, a similar spectral approach is taken to calculate the development of the wake flowfield behind a symmetric cascade of bodies for steady two-dimensional laminar incompressible flow at large Reynolds number. The method is applied to the separated flow past a cascade of finite thickness flat plates.

The spectral or integral relations methods are well-documented techniques for the solution of the laminar incompressible boundary-layer equations for attached flows.[11,12] No evidence could be found in the literature of the failure of these techniques (for attached flow) due to the presence of a singularity in the resulting matrix equations. It is noted that in both Kumar and Yajnik[10] (see Kumar[13]) and the present analysis, singularities occur which limit successful solutions to those flows with "moderate" recirculation zones and "limited" detail. This behavior is herein attributed to the use of the parabolic slender channel equations to model incompressible flows with recirculation. Further details appear in the body of the paper.

Limit Equations

Consider a steady two-dimensional laminar incompressible flow that is confined in the transverse direction so that the transverse length scale is of order one. Use the coordinate system of Fig. 1. x and y are nondimensionalized by the "channel" half width and velocities by the characteristic velocity of the downstream flow. For the problem of Kumar and Yajnik,[10] $y = \pm 1$ are channel walls. In our case, the boundaries are symmetry boundaries or "inviscid walls." The Navier-Stokes equation in nondimensional stream function

Presented as Paper 81-0055 at the AIAA 19th Aerospace Sciences Meeting, St. Louis, Mo., Jan. 12-15, 1981; submitted March 10, 1981; revision received Aug. 3, 1981.

*Professor, Department of Aerospace Engineering. Associate Fellow AIAA.

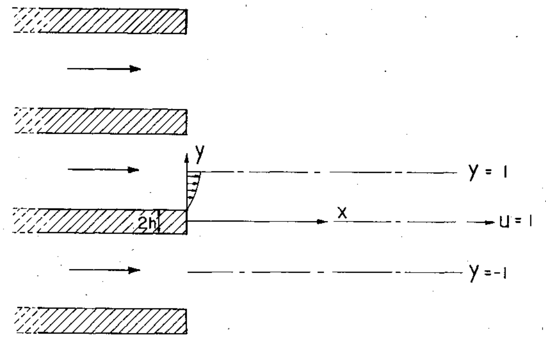

Fig. 1 Flow configuration and coordinate system.

form is

$$\Psi_y \nabla^2 \Psi_x - \Psi_x \nabla^2 \Psi_y = R^{-1} \nabla^4 \Psi \tag{1}$$

where Ψ is the stream function and R is the Reynolds number.

Consider first the rationale of Kumar and Yajnik[10] for the choice of the appropriate high Reynolds number limit of Eq. (1). Represent the streamwise and transverse length scales by L_x and L_y and $L_y \sim 1$. For the bulk of the flow region (most of the recirculation zone and the far wake) the streamwise velocity component is of order one. For large values of the Reynolds number, there are two principal limits of Eq. (1) corresponding to L_x of order one and L_x of the order of R.

For $L_x \sim 1$, the Euler equation for inviscid flow is obtained

$$\Psi_y \nabla^2 \Psi_x - \Psi_x \nabla^2 \Psi_y = 0 \tag{2}$$

and for $L_x \sim R$, the following parabolic viscous equation is obtained:

$$\Psi_y \Psi_{yyx} - \Psi_x \Psi_{yyy} = R^{-1} \Psi_{yyyy} \tag{3}$$

Note that this is the Prandtl boundary-layer equation[14] with different scaling (the standard boundary-layer equation has $L_x \sim 1$, $L_y \sim R^{-1/2}$). In addition, the transverse velocity component is of the order of R^{-1} and the pressure is of order one. The usual third-order form of the boundary-layer equation is recovered by an integration of Eq. (3) with respect to y and an identification of the streamwise pressure gradient as the constant of integration. The transverse pressure gradient is zero.

The validity of the aforementioned principal limits is reinforced in the study of the asymptotic downstream solution for the channel flow. For large values of the Reynolds number, the Navier-Stokes equation (1) linearized about Poiseuille flow yields two sequences of eigenvalues which correspond to $L_x \sim 1$ and $L_x \sim R$. Kumar and Yajnik[10] postulate two classes of internal separated flows corresponding to the two limits and only study those flows where $L_x \sim R$ is appropriate. This scaling is further justified by the demonstration that the Reynolds number is the correct streamwise length scale for the recirculation zone in a number of numerical solutions of the Navier-Stokes equations.

The symmetric cascade problem has the same geometry as the base in a channel flow of Kumar and Yajnik but with inviscid walls and it is expected that the basic character of the flows is similar. The similarity can be reinforced by the study of the linearized far wake solution. Let

$$\Psi = y + \psi \tag{4}$$

and substitute into Eq. (1) and linearize. The governing equations reduce to the Oseen equation

$$\left(\nabla^2 - R\frac{\partial}{\partial x}\right)\nabla^2 \psi = 0 \tag{5}$$

The boundary conditions on the symmetry boundaries are

$$\psi = \psi_{yy} = 0 \quad \text{on} \quad y = \pm 1 \tag{6}$$

The eigenfunction solution is

$$\psi = e^{-\lambda_m x} \sin m\pi y \tag{7}$$

with two sets of eigenvalues

$$\lambda_m = \pm m\pi \tag{8a}$$

$$= -\frac{R}{2} \pm \frac{R}{2}\left[1 + \frac{4\pi^2 m^2}{R^2}\right]^{1/2} \tag{8b}$$

with m an integer.

The first set corresponds to $L_x \sim 1$ and the Euler equation. As $R \to \infty$, the second set approaches $\lambda_m \sim \pi^2 m^2/R$, $-R$ and, therefore, for downstream decay $\lambda_m \sim R^{-1}$ and $L_x \sim R$. This has essentially the same character as the channel flow.

Problem Formulation and Method of Solution

Consider the steady two-dimensional laminar incompressible large Reynolds number flow past a symmetric cascade of finite thickness flat plates as shown in Fig. 1. Lengths have been normalized by half the plate separation distance and velocities by the stream velocity. h is the ratio of plate thickness to plate separation. It is intended to calculate the development of the wake from the base of the plate to downstream infinity.

The limit Eq. (3) is valid except in the immediate neighborhood of the base with $L_x \sim 1$. If the contracted streamwise coordinate $X = x/R$ is used, Eq. (3) becomes

$$\Psi_{yyyy} = \Psi_y \Psi_{yyX} - \Psi_X \Psi_{yyy} \tag{9}$$

Note that the pressure, which is an unknown in this confined flow, does not appear in Eq. (9). It will be determined with the use of an integral momentum equation. In previous analyses,[8,9] primitive variables were used and the pressure was determined simultaneously with the velocity field. The transverse boundaries are symmetry boundaries with

$$\Psi = \pm 1, \quad \Psi_{yy} = 0 \qquad y = \pm 1 \tag{10}$$

Note that Eq. (9) is first order in X whereas the original Navier-Stokes Eq. (1) is fourth order. Therefore only one boundary condition can be applied at $X = 0$ and, in general, this would come from matching with a local or inner solution valid in the neighborhood of the base. In the contracted coordinate X, the streamwise extent of the local solution essentially vanishes.

The initial condition used in this study is

$$\Psi(0,y) = \Psi_0(y) \tag{11}$$

which is equivalent to prescribing the initial streamwise velocity profile $u_0(y) = \Psi_{0y}$. The slope of the streamlines cannot be prescribed at the initial station.

Kumar and Yajnik[10] use an eigenfunction expansion method which is a variation of integral relations. The expansion functions for their problem must be determined numerically which appears to present a serious obstacle to the calculation of more than five terms in the expansion and the subsequent establishment of more positive evidence of convergence.

Their method is also a spectral or Galerkin method (see Orszag and Gottlieb[11]) with the streamwise coordinate as the time-like variable. Orszag and co-workers have recently been developing the technique to study the solution of the boundary-layer equations for attached flow.[15] Since the boundary conditions [Eq. (10)] are periodic, a Fourier series expansion can be used and the stream function is taken to be

$$\Psi = y + \sum_{1}^{N} a_n(X)\sin n\pi y \tag{12}$$

where the series is truncated after N terms. Equation (12) is substituted into Eq. (9) and after the result is multiplied by $\sin m\pi y$ and integrated from $y=-1$ to $+1$ the following set of coupled quasilinear ordinary differential equations is obtained:

$$a_m' + m^2\pi^2 a_m = \sum_n \sum_q C_{mnq} a_n' a_q$$

$$= \frac{\pi}{2m}\Big\{ \sum_{n=1}^{m-1} (m-n)(m-2n) a_n' a_{m-n}$$

$$- \sum_{n=m+1}^{N} (m-n)(m-2n) a_n' a_{n-m}$$

$$- \sum_{n=1}^{N-m} (m+n)(m+2n) a_n' a_{n+m} \Big\} \quad m=1,N \tag{13}$$

where the prime denotes differentiation with respect to X. The initial values of the functions $a_m(X)$ are given by

$$a_{m0} = a_m(0) = \frac{2}{\pi m}\int_0^1 u_0(y)\cos m\pi y \, dy \tag{14}$$

Closed-form solutions of Eq. (13) can be found for $N=1$ and 2. They are

$$N=1: \quad a_1/a_{10} = e^{-\pi^2 X}$$

$$N=2: \quad \frac{a_1}{a_{10}} = \left[\frac{1+3\pi a_{20}}{1+3\pi a_2}\right]^{3/4} e^{-\pi^2 X}$$

$$a_2/a_{20} = e^{-4\pi^2 X} \tag{15}$$

Note that for these two cases $a_m(X) \sim \exp(-m^2\pi^2 X)$ for $X \to \infty$ which is consistent with the results from the linearized Eqs. (8). Also, the solution for $N=1$ was obtained by Kovasznay[16] as a model for the flow past a two-dimensional grid and is an exact solution of the Navier-Stokes Eq. (1).

Once the stream function is found from the solution of Eqs. (13) and (14), the pressure field can be obtained as follows.

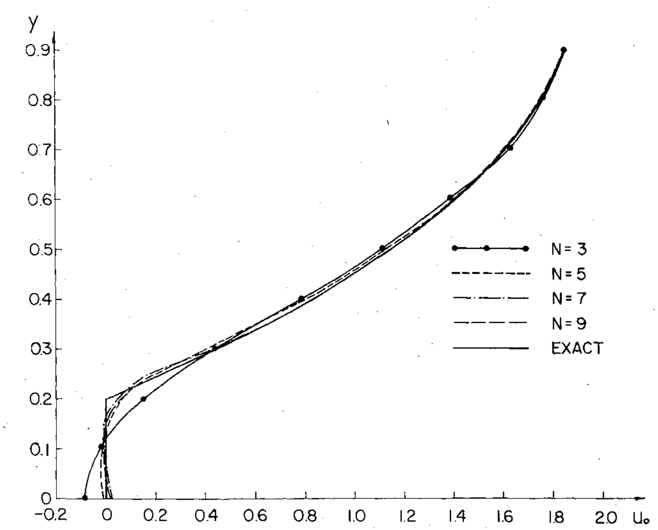

Fig. 2 Representation of parabolic initial profile for $h=0.2$.

Equation (9) can be integrated once with respect to y to yield

$$\Psi_{yyy} + \Psi_X \Psi_{yy} - \Psi_y \Psi_{Xy} = p'(X) \tag{16}$$

where the constant of integration is identified as the streamwise pressure gradient since this is the standard form of the boundary-layer equations. The pressure in Eq. (16) has been made nondimensional by the product of the density and the square of the stream velocity. Equation (16) is integrated from $y=0$ to 1 to yield

$$(\Psi_{yy} + \Psi_X \Psi_y)\Big|_0^1 = 0 = \frac{d}{dX}\int_0^1 (p + \Psi_y^2)\,dy \tag{17}$$

where Eq. (10) and the wake centerline symmetry conditions have been used. Equation (17) is integrated with respect to X and if the mean pressure P is defined as

$$P = \int_0^1 p\,dy$$

and the subscript 0 refers to $X=0$, then the following integral momentum equation is obtained:

$$P - P_0 = \int_0^1 (u_0^2 - u^2)\,dy \tag{18}$$

Results and Discussion

Strictly speaking, the problem represented by the ordinary differential equation set Eqs. (13) with the initial conditions of Eq. (14) is the development of the laminar wake downstream of an initial station with a given streamwise velocity profile. The flow configuration upstream of this initial station enters the problem only through the initial conditions. Therefore the choice of an appropriate initial profile is critical to the modeling.

It is assumed that the flow between the plates is fully developed upstream of the base. A parabolic initial profile (as in Kumar and Yajnik[10]) is chosen at the base as

$$u_0(y) = \frac{3}{1-h}\left[\frac{y-h}{1-h} - \frac{1}{2}\left(\frac{y-h}{1-h}\right)^2\right] \quad h \le y \le 1$$

$$= 0 \qquad\qquad 0 \le y \le h \tag{19}$$

For this profile, the maximum velocity, $u_0(1)$, is $1.5/(1-h)$. The momentum flux is

$$\int_0^1 u_0^2\,dy = \frac{1.2}{(1-h)}$$

The initial values of the Fourier coefficients are

$$a_{m0} = \frac{1}{\pi^4 m^4 (h-1)^3}[\sin m\pi h + m\pi(1-h)\cos m\pi h] \tag{20}$$

Note that $a_{m0} \sim m^{-3}$. The representation of the initial profile for $h=0.2$ is shown in Fig. 2 as a function of the number of terms N.

The wake centerline velocity is obtained from Eq. (12) as

$$u_c(X) = 1 + \sum_{1}^{N} n\pi a_n(X) \tag{21}$$

The length of the recirculation zone X_R is determined from

$$u_c(X_R) = 0 \tag{22}$$

Equation (13) can be written in the form

$$\sum_n \left(\delta_{mn} - \sum_q C_{mnq} a_q\right) a_n' = -m^2\pi^2 a_m$$

which can be expressed as the matrix equation (see Ref. 10)

$$\boldsymbol{D a}' = -\boldsymbol{\Lambda a} \tag{23}$$

where $\boldsymbol{a}$ is a column matrix with elements $a_1, a_2, \ldots, a_N$ and $\boldsymbol{D}$ and $\boldsymbol{\Lambda}$ are $N \times N$ square matrices. At a given station X, the derivative $\boldsymbol{a}'$ can be calculated if $\boldsymbol{a}$ is known provided the determinant det $\boldsymbol{D}$ is nonzero. For $N \geq 3$, the ordinary-differential equation set Eq. (23) is integrated numerically using a fourth-order variable step size Runge-Kutta-Fehlberg method.

Kumar and Yajnik[10] state that only a few terms ($N=3,5$) are necessary to establish the gross flow features and that additional terms are needed only to supply detail in the region near $X=0$. They use at most five terms in their analysis and present solutions for the complete range of h.

An important part of the goal of this research is to study in more detail the numerical convergence properties of the solution. In this regard the wake centerline velocity is shown as a function of N ($N \leq 8$) for $h=0.2$ in Fig. 3. It is noted that the solutions for $N=6$ and 8 essentially merge before the recirculation zone closes and only differ significantly for $X<0.002$. A good approximation for the length of the recirculation zone X_R can be obtained with $N=6$.

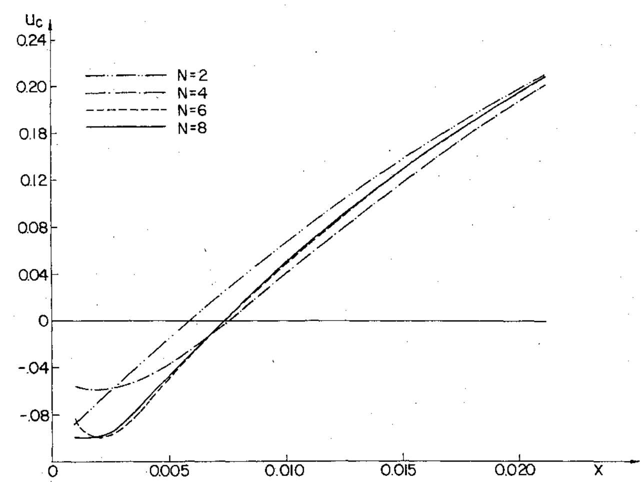

Fig. 3 Wake centerline velocity for $h=0.2$.

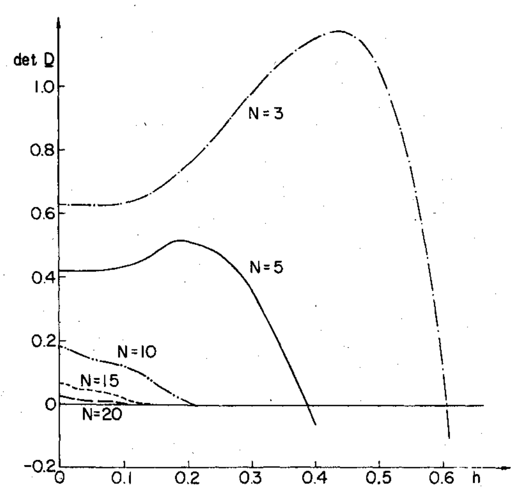

Fig. 4 Determinant of coefficient matrix at initial station ($X=0$) for parabolic initial profile.

When the preceding calculation for $h=0.2$ was attempted for $N=9$, det $\boldsymbol{D}$ became zero before one spatial integration step could be completed. The integration procedure was used to obtain solutions for values of h in steps of 0.1 and it was discovered that for a given value of h, a maximum number of terms N could be computed beyond which the identical singular behavior would occur. This maximum number decreases with increasing h and for $h>0.5$, the only solution that can be calculated is the closed-form one for $N=2$ [Eq. (15)]. The maximum numbers N for $h=0.3$, 0.4, 0.5 are 5, 4, and 3, respectively.

This singular behavior can be anticipated as can be seen from Fig. 4 which is a plot of the determinant det $\boldsymbol{D}$ of the coefficient matrix of the system of equations Eq. (23) vs h for the initial values of the coefficients and a fixed N. For a fixed value of N, it is seen that the determinant approaches zero with a steep slope for a value of h which decreases with increasing N. For a fixed value of N, it would appear that a

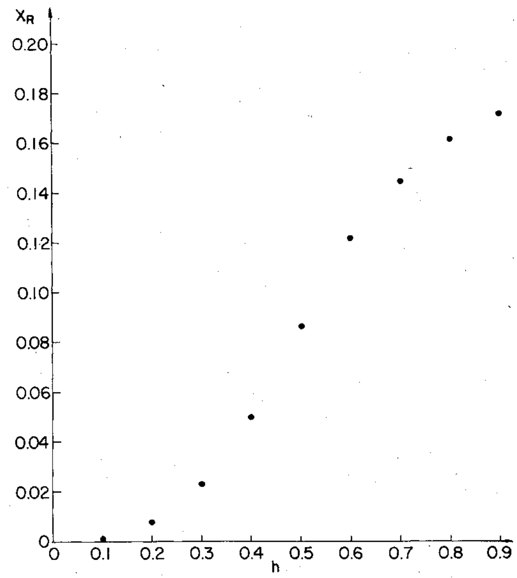

Fig. 5 Length of recirculation zone.

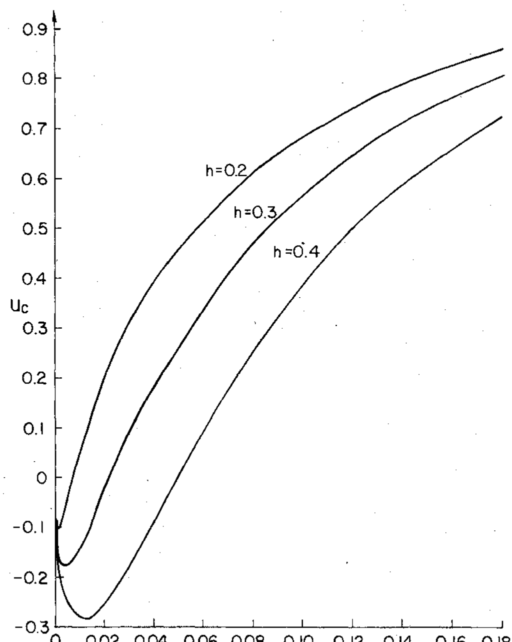

Fig. 6 Wake centerline velocity for $h=0.2$, 0.3, and 0.4 (best approximation).

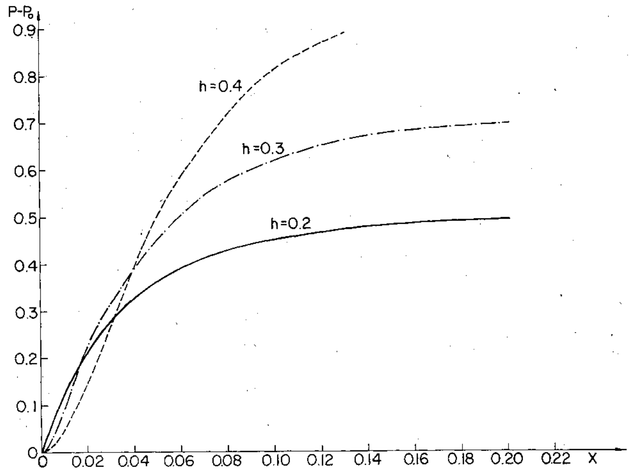

Fig. 7 Mean pressure distribution for $h=0.2$, 0.3, and 0.4 (best approximation).

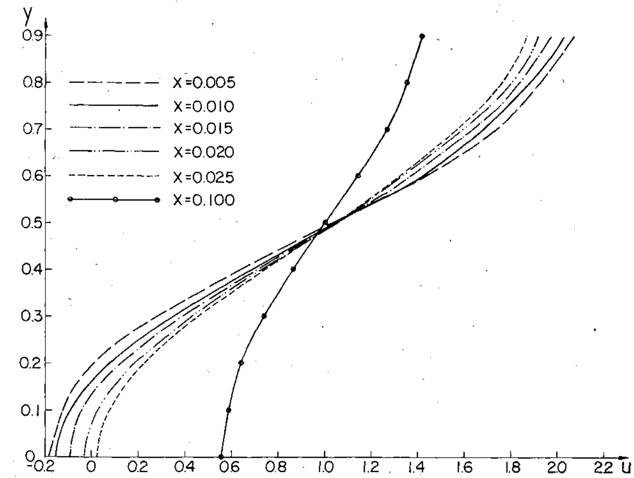

Fig. 8 Streamwise velocity profiles for $h=0.3$.

solution is possible for a value of h greater than the one for which the determinant is zero. Calculations, however, showed that although the integration did in fact start, the determinant always vanished before the recirculation zone closed. Similar singular behavior occurred in the calculation of Kumar and Yajnik[10] (reported in Kumar[13]) although in a few of their cases, for a fixed geometry, singular behavior for one value of N did not preclude a successful calculation with a larger value of N.

For small values of h, certainly for $h \leq 0.2$ when $N \geq 8$, the solution appears to be converging to a physically reasonable flowfield. For values of $h \leq 0.5$ when at least three terms in the expansion can be calculated, the results appear to be reasonable approximations to the gross flowfield downstream of the base and especially in the far wake. It is noted that Kumar and Yajnik[10] report good agreement with numerical solutions of the Navier-Stokes equations for flow variables such as X_R with $N=3$ and 5.

The best approximation for the length of the recirculation zone X_R is given vs h in Fig. 5. The wake centerline velocity and pressure distribution for $h=0.2$, 0.3, and 0.4 are shown in Figs. 6 and 7. A set of velocity profiles for $h=0.3$ is shown in Fig. 8 for both reverse and forward flow.

It has been stated that the choice of the initial streamwise velocity profile is critical to the modeling of the flow upstream of the base. It is for this reason that an alternate initial profile, one on the verge of separating, will also be considered. The cubic "separating" profile is given by

$$u_0(y) = \frac{1}{1-h}\left[6\left(\frac{y-h}{1-h}\right)^2 - 4\left(\frac{y-h}{1-h}\right)^3\right] \qquad h \leq y \leq 1$$

$$= 0 \qquad 0 \leq y \leq h \quad (24)$$

For this profile, $u_{0y}(h)=0$, the maximum velocity is $2/(1-h)$ and the momentum flux is

$$\int_0^1 u_0^2 \mathrm{d}y = \frac{52}{35}\frac{1}{1-h}$$

The initial values of the Fourier coefficients are

$$a_{m0} = \frac{24}{\pi^5 m^5 (1-h)^4}[2\cos m\pi - 2\cos m\pi h$$

$$+ m\pi(1-h)\sin m\pi h] \quad (25)$$

Note that $a_{m0} \sim m^{-4}$. The cubic profile is smoother than the parabolic profile at $y=h$ (its lowest order discontinuous derivative is the second) and therefore should provide better solution convergence properties as well as a better representation of the initial profile for a given number of terms.

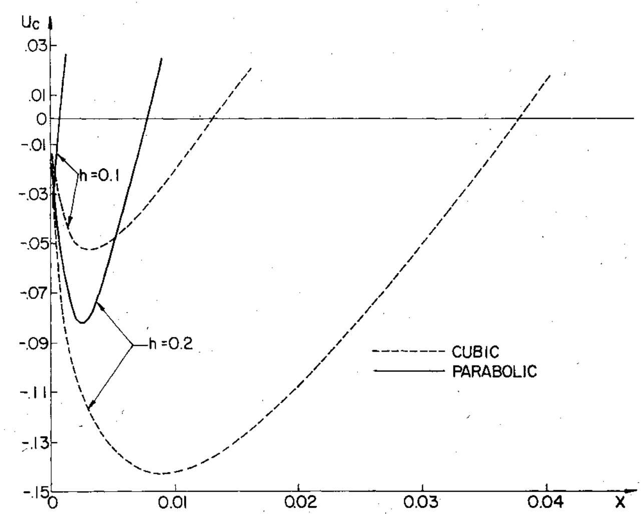

Fig. 9 Wake centerline velocity for $h=0.1$ and 0.2 for parabolic and cubic initial profiles.

The solution with the cubic initial profile displays the same qualitative singular behavior as that observed with the parabolic one. The solutions appear to be converging more rapidly with N but the maximum number of terms that can be calculated is always less with the cubic profile. The wake centerline velocity is plotted in Fig. 9 for $h=0.1$ and 0.2 for both initial profiles. The curves are qualitatively similar but the gross features such as X_R differ by several hundred percent with the cubic profiles leading to longer recirculation zones.

It appears that the cubic and parabolic profiles must correspond to different upstream geometries. To aid in the determination of which profile is more appropriate to represent the flow configuration of Fig. 1, the streamline patterns (including the recirculation zone boundary $\Psi=0$) for $h=0.2$ are plotted in Fig. 10 for both profiles. It is clear from the figure that the parabolic initial profile more closely models the expected streamline pattern and initial streamline slopes, especially near the base. A partial explanation for the significant initial profile influence upon the solution is that the confined nature of the flow requires a large shift in the initial elevation of corresponding streamlines as can be seen in Fig. 10.

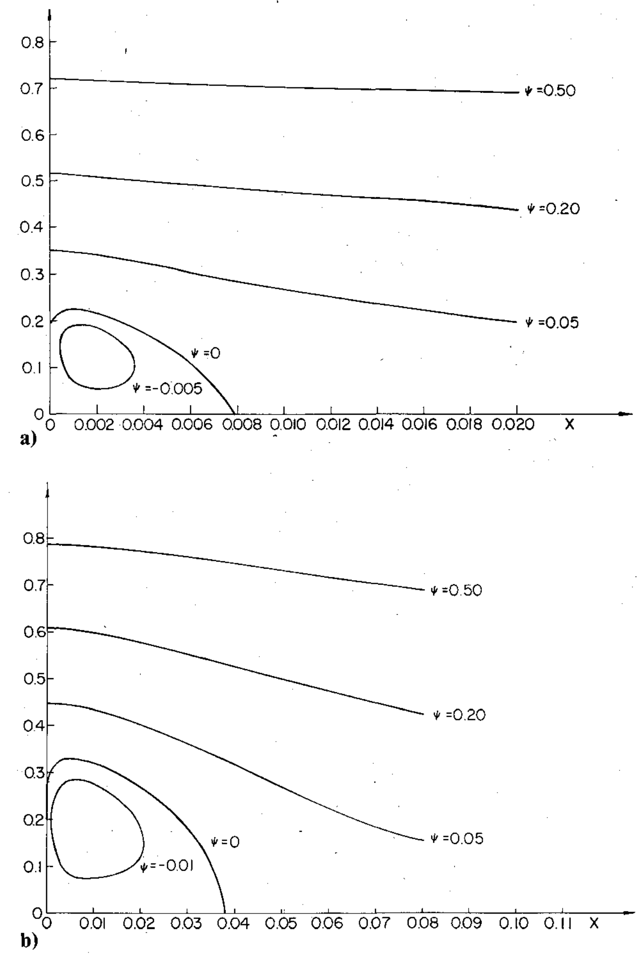

Fig. 10 Streamlines for $h = 0.2$; a) parabolic initial profile, b) cubic initial profile.

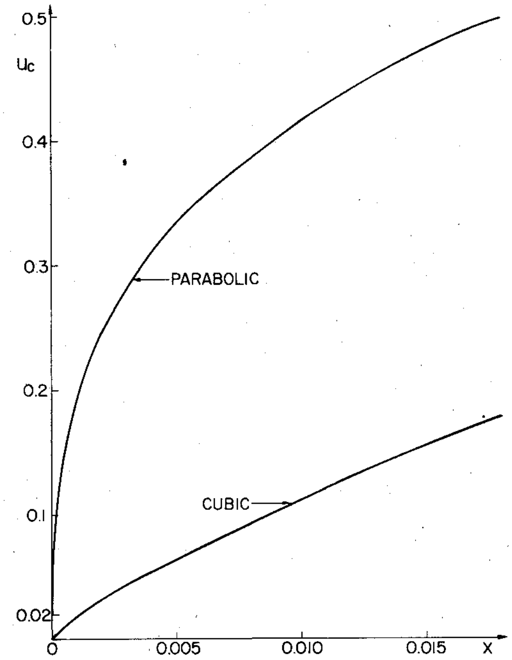

Fig. 11 Wake centerline velocity for flat plate ($h = 0$) for parabolic and cubic initial profiles.

An interesting limiting case for the problem under consideration occurs when the ratio of plate thickness to plate separation approaches zero. This is the flow past a cascade of infinitely thin flat plates. This problem possesses no recirculation zone and no reversed flow. More than thirty terms can be calculated with the parabolic initial profile and more than fifteen can be calculated with the cubic profile. The wake centerline velocity is plotted in Fig. 11 for both initial profiles. It is expected that the parabolic initial profile is appropriate for this limiting case.

Conclusions

A spectral method has been investigated for use in the calculation of the development of the wake flowfield behind a symmetric cascade-like arrangement of bodies for steady two-dimensional laminar incompressible flow at large Reynolds number. This is a companion study to the research of Kumar and Yajnik[10] who consider channel flow and also solve the stream function version of the slender channel equations.

The results of our analysis tend to temper somewhat the overwhelmingly positive results presented by Kumar and Yajnik.[10] First of all, several term solutions have only been obtained for flows with limited recirculation regions. For plates occupying more than half of the transverse extent of the flow region, no more than two terms in the series expansion for the stream function can be calculated. Second, for a given plate thickness to plate separation ratio, there is a limit to the number of terms that can be calculated. There is, therefore, a limit to the detail or accuracy of the solution that can be obtained. Third, the solution is seen to possess a strong dependence on the initial streamwise velocity profile. In general, the solution should be considered to model the downstream wake development from an initial streamwise velocity profile without the assumption of a particular upstream geometry.

In an attempt to assess the reasons for the just noted singular behavior, it is important to distinguish between the choice of the governing equations and of the method for their solution. As stated in the Introduction, the success of the spectral or integral relations method is well documented for the solution of the laminar incompressible boundary-layer equations for attached flow and the author is not aware of reported evidence of singular behavior.

It seems more likely that the singular behavior is attributable to the use of the parabolic slender channel equations to model incompressible flows with recirculation (whose character is elliptic when reversed flow is present). In fact, this same type of behavior has been reported for finite-difference solutions to the same parabolic partial-differential equations. For example, Blottner[8] reports a stable numerical solution for a channel flow calculation with separation for one value of the streamwise grid spacing which could not be repeated as the grid size was reduced.

As shown in Kumar and Yajnik[10] and reported here, physically reasonable solutions have been obtained with a relatively small number of terms in the stream function expansion. The results of Ref. 10 compare well to more detailed numerical solutions of the Navier-Stokes equations. For plate thickness to separation ratios of less than 20%, the gross features of the wake flowfield for the cascade-like arrangement of finite thickness flat plates appear to be described quite adequately with solutions possessing reasonable convergence properties. Even for larger values of the ratio, approximate solutions are available.

The numerical solution of the ordinary differential equation set can be obtained using integration routines available in most computer libraries. No special attention is necessary to enable the calculation to proceed successfully through a recirculation zone.

The importance of the initial streamwise velocity profile is magnified in flows that are confined in the transverse direction. For the problem investigated by Kumar and Yajnik[10] and this author, when the flow far upstream of the base or initial station is a parabolic channel flow, the appropriate initial streamwise velocity profile that models the upstream geometry reasonably well appears to be parabolic.

In conclusion, when boundary-layer-like parabolic equations are used to describe the high Reynolds number internal separated flows studied here and in Ref. 10, the spectral method possesses some clear advantages over other techniques. One way to help pinpoint the cause of the singular behavior would be to use the preceding spectral method for the solution of the complete large Reynolds number Navier-Stokes equations. The resulting ordinary differential equation set would be fourth order in X and boundary conditions would need to be specified at a downstream station.

Acknowledgments

This research was performed while the author was an ASEE-Navy Summer Faculty Research Fellow in the Applied Mathematics Branch of the Naval Surface Weapons Center in White Oak, Maryland. The author would like to thank the members of the Branch, especially Drs. Jay Solomon and John Bell, for their help during the course of the research.

References

[1] Davis, R. T., "Numerical Solution of the Incompressible Navier Stokes Equations for Two Dimensional Flows at High Reynolds Number," *Proceedings of the First International Conference on Numerical Ship Hydrodynamics,* National Bureau of Standards, Oct. 1975, pp. 351-383.

[2] Davis, R. T. and Werle, M. J., "Numerical Methods for Interacting Boundary Layers," *Proceedings of the 1976 Heat Transfer and Fluid Mechanics Institute*, Univ. of California, Davis, June 1976.

[3] Stewartson, K., "Multistructured Boundary Layers on Flat Plates and Related Bodies," *Advances in Applied Mechanics*, Vol. 14, 1974, pp. 145-239.

[4] Sychev, V. V., "On Laminar Separation," *Fluid Dynamics*, Vol. 7, No. 3, March-April 1974, pp. 407-417.

[5] Smith, F. T., "Laminar Flow of an Incompressible Fluid Past a Bluff Body: The Separation, Reattachment, Eddy Properties and Drag," *Journal of Fluid Mechanics*, Vol. 92, Pt. 1, 1972, pp. 171-205.

[6] Cebeci, T., Keller, H. B., and Williams, P. G., "Separating Boundary-Layer Flow Calculations," *Journal of Computational Physics*, Vol. 31, No. 3, June 1979, pp. 363-378.

[7] Williams, J. C., "Viscous Compressible and Incompressible Flow in Slender Channels," *AIAA Journal*, Vol. 1, Jan. 1963, pp. 186-195.

[8] Blottner, F. G., "Numerical Solution for Slender Channel Laminar Flows," *Computer Methods in Applied Mechanics and Engineering*, Vol. 11, 1977, pp. 319-339.

[9] Smith, F. T. and Eagles, P. M., "The Influence of Nonparallelism in Channel Flow Stability," *Journal of Engineering Mathematics*, Vol. 14, No. 3, 1980, pp. 219-237.

[10] Kumar, A. and Yajnik, K. S., "Internal Separated Flows at Large Reynolds Number," *Journal of Fluid Mechanics*, Vol. 97, Pt. 1, 1980, pp. 27-51.

[11] Orszag, S. A. and Gottlieb, D., *Numerical Analysis of Spectral Methods: Theory and Applications*, SIAM, 1977.

[12] Holt, M., *Numerical Methods in Fluid Dynamics*, Springer-Verlag, Berlin, 1977, Chap. 5.

[13] Kumar, A., "Analysis and Calculation of Internal Separated Flow at Large Reynolds Number," Ph.D. Thesis, Department of Mechanical Engineering, Indian Institute of Technology, Kanpur, June 1976.

[14] Van Dyke, M., *Perturbation Methods in Fluid Mechanics*, The Parabolic Press, 1975, p. 127.

[15] Orszag, S. A., "Spectral Methods for Problems in Complex Geometries," *Journal of Computational Physics*, Vol. 37, No. 1, Aug. 1980, pp. 70-92.

[16] Kovasznay, L.I.G., "Laminar Flow Behind a Two-Dimensional Grid," *Proceedings of the Cambridge Philosophical Society*, Vol. 44, 1948, pp. 58-62.